Bolted Joint and Preload Stuff
11-1-04

Figure 1 shows a standard through-hole bolted joint. The bolt can be modeled as a spring, with stiffness k_b. The upper and lower bolted members can also be modeled as springs with respective stiffnesses k_1 and k_2. At its equilibrium length, a spring applies no axial force. The equilibrium lengths of springs 1, 2 and b are defined as $L_{1,e}$, $L_{2,e}$, and $L_{b,e}$, respectively. The spring forces can be expressed as follows:

$$f_b = k_b \left(L_b - L_{b,e} \right) \tag{1.1}$$

$$f_1 = k_1 \left(L_1 - L_{1,e} \right) \tag{1.2}$$

$$f_2 = k_2 \left(L_2 - L_{2,e} \right) \tag{1.3}$$

where the sign convention is positive for tension and negative for compression.

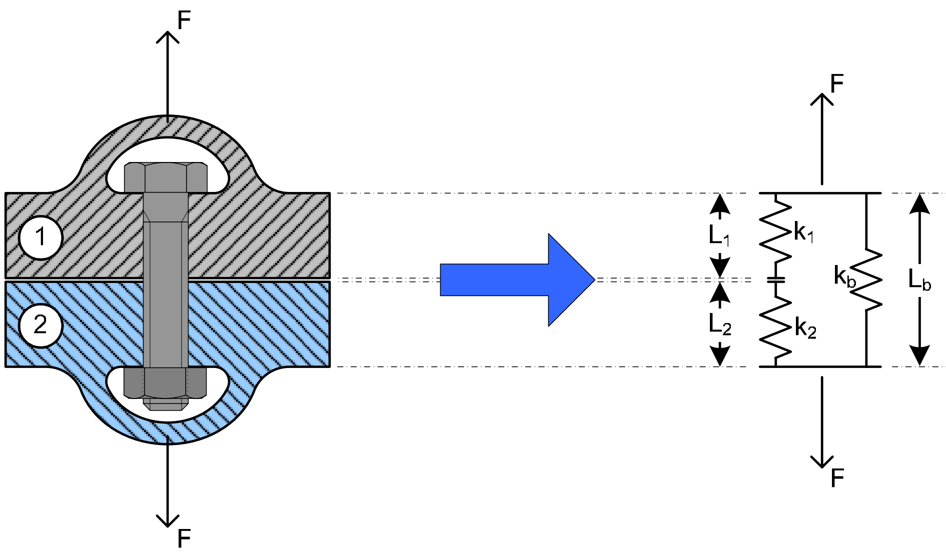

Figure 1. Bolted joint

Note from the figure that the spring models of the clamped members are only applicable when the clamped members are in compression, i.e., when their lengths (L_1, L_2) are shorter than their equilibrium lengths ($L_{1,2}$, $L_{2,e}$). Physically, this means that the normal force between the surfaces of members 1 and 2 can only be compressive, which is intuitively obvious. From here on, we will assume compressive loads only.

From the figure, we can observe the following relation among the three spring lengths:

$$L_b = L_1 + L_2 \tag{1.4}$$

We can also write two equations relating the applied force, F, and spring forces:

$$F = f_b + f_2 \tag{1.5}$$

$$f_1 = f_2 \tag{1.6}$$

Plugging (1.1) thru (1.3) into (1.5), we obtain:

$$k_b L_b + k_2 L_2 = F + k_b L_{b,e} + k_2 L_{2,e} \tag{1.7}$$

And plugging (1.2) and (1.3) into (1.6), we get:

$$k_1 L_1 - k_2 L_2 = k_1 L_{1,e} - k_2 L_{2,e} \tag{1.8}$$

Equations (1.4), (1.7), and (1.8) comprise a system of three equations with three unknowns, and can be expressed in standard matrix form:

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & k_2 & k_b \\ k_1 & -k_2 & 0 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_b \end{bmatrix} = \begin{bmatrix} 0 \\ F + k_b L_{b,e} + k_2 L_{2,e} \\ k_1 L_{1,e} - k_2 L_{2,e} \end{bmatrix} \tag{1.9}$$

The solution to this system is:

$$L_1 = \frac{k_1 k_b L_{1,e} + k_1 k_2 L_{1,e} + k_2 F + k_2 k_b \left(L_{b,e} - L_{2,e} \right)}{k_b \left(k_1 + k_2 \right) + k_1 k_2} \tag{1.10}$$

$$L_2 = \frac{k_2 k_b L_{2,e} + k_1 F + k_1 k_b \left(L_{b,e} - L_{1,e} \right) + k_1 k_2 L_{2,e}}{k_b \left(k_1 + k_2 \right) + k_1 k_2} \tag{1.11}$$

$$L_b = \frac{k_1 k_2 \left(L_{1,e} + L_{2,e} \right) + \left(k_1 + k_2 \right) F + k_1 k_b L_{b,e} + k_2 k_b L_{b,e}}{k_b \left(k_1 + k_2 \right) + k_1 k_2} \tag{1.12}$$

And these expressions can be plugged back into (1.1) and (1.3) to yield the forces in the bolt and clamped members as a function of the applied force, *F*:

$$f_b = \frac{k_b \left(k_1 + k_2 \right)}{k_b \left(k_1 + k_2 \right) + k_1 k_2} F + \frac{k_1 k_2 k_b \left(L_{1,e} + L_{2,e} - L_{b,e} \right)}{k_b \left(k_1 + k_2 \right) + k_1 k_2} \tag{1.13}$$

$$f_2 = \frac{k_1 k_2}{k_b \left(k_1 + k_2 \right) + k_1 k_2} F - \frac{k_1 k_2 k_b \left(L_{1,e} + L_{2,e} - L_{b,e} \right)}{k_b \left(k_1 + k_2 \right) + k_1 k_2} \tag{1.14}$$

For the special case where $k_1 = k_2$ and $L_{1,e} = L_{2,e}$, these formulas simplify to:

$$f_b = \frac{2k_b}{2k_b + k_1} F + \frac{k_1 k_b \left(2L_{1,e} - L_{b,e} \right)}{2k_b + k_1} \tag{1.15}$$

$$f_2 = \frac{k_1}{2k_b + k_1} F - \frac{k_1 k_b \left(2L_{1,e} - L_{b,e} \right)}{2k_b + k_1} \tag{1.16}$$

In both the special and general cases, the forces in the bolt and clamped members are seen to vary linearly with the applied force, *F*. With no applied force (*F=0*), the forces in the bolt and clamped members are equal and opposite, as would be expected. Also note the slopes in both (1.13) and (1.14) are positive (for positive spring constants). This means when *F* increases in the positive direction, so do the forces in the bolt and clamped members. In the case of the bolt which begins in tension (for *F=0*), the tensile force increases as *F* increases. However, the clamped members begin under compression, meaning f_2 is negative. Therefore an increase in *F* moves this compressive force closer to zero, decreasing its magnitude.

For the more special case where the clamped members are much stiffer than the bolt ($k_1=k_2>>k_b$), (1.15) and (1.16) become:

$$f_b = k_b \left(2L_{1,e} - L_{b,e} \right) \tag{1.17}$$

$$f_2 = F - k_b \left(2L_{1,e} - L_{b,e} \right) \tag{1.18}$$

For this case, the tensile force in the bolt is independent of the applied force, F. The compressive force in the clamped members decreases (or increases algebraically) with increasing F until the clamping force reaches zero. Any further increase in F results in separation of the clamped surfaces, which this model does not take into account.